PROTON-PROTON SCATTERING AT HIGH ENERGIES

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ABSTRACT. In this paper the differential cross sections for proton-proton scattering have been calculated in the Born's approximation using potentials consisting of the tensor operator multiplied respectively by $\frac{e^{-\lambda r}}{\lambda r^2}$ and $\frac{e^{-\lambda r}}{\lambda^2 r^3}$. Results for the second potential have been shown to give marked angular isotropy which agree qualitatively better with the experimental results of proton-proton scattering at 345 MeV energy than the existing calculations made with potentials derivable from field theories.

INTRODUCTION

Chamberlain *et al* (1951) have recently determined proton-proton scattering cross sections at high energies. Their results for 345 MeV show that the differential cross section between 90° and 15° is very much independent of the angles having the value 3.8×10^{-27} cm^2/steradian (centre of mass system). Such spherically symmetric scattering could qualitatively be explained by saying that the scattering is due to an interaction in the S state. But this is not a possible explanation of the observed results because the cross sections, as calculated on the basis of an interaction in the S state, are too low in comparison with the experimentally determined values.

So far, it has not been possible to explain even qualitatively the observed isotropic scattering in the above-mentioned case with the help of potentials as obtained from field theoretical considerations. Some workers have, however, tried to approach the problem in a phenomenological manner. Case and Pais (1951) have attempted to explain the observed angular isotropy by introducing a spin-orbit coupling in the nuclear interaction. Christian and Noyes (1950) have used a combination of central and tensor potentials. The central potential is a square well of range 2.6×10^{-13} cms. and the tensor potential is of the from $V_t \frac{e^{-r/R}}{(r/R)^2}$ with $R = 1.6 \times 10^{-13}$ cms.; $V_t = \pm 18$ MeV. Their curve, however, rises steadily and rapidly below 40°. They also have plotted a curve for which the tensor term is of the Yukawa type and also another curve for no tensor potential. Their curves clearly show that the tensor term not only flattens the curve, but also raises the

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value of the scattering cross sections at large angles. Furthermore, the singular tensor curve $\left(\frac{1}{r^2}\text{-singularity}\right)$ is flatter than the Yukawa tensor curve. In the present paper, the central term has been omitted altogether, thus allowing scattering to take place in the triplet states only. Furthermore, a potential with $\frac{1}{r^3}$-singularity has been tried to investigate the effect of radial dependence on the cross section. As will be seen, the term with $\frac{1}{r^2}$- singularity shows considerable variation of the differential cross section with angle; and hence it is not likely that the Yukawa tensor potential will give isotropic scattering. Hence it has not been tried. Jastrow (1951) has attempted to explain the high energy proton-proton scattering taking an interaction consisting of a hard core of finite radius in the singlet state in addition to an exponential radial dependence outside the core; and a triplet potential consisting of tensor and exchange terms in addition to an exponential radial dependence. So far, his calculations seem to fit the experimental data best. The existence of a hard core of finite radius is, however, questionable on the point of view of relativity.

In this paper, proton-proton scattering cross sections have been calcutated in the centre of mass system taking the potentials

$$V_1=\frac{f^2}{4\pi}\,.\,S_{12}.\,\frac{e^{-\lambda r}}{\lambda r^2} \qquad \dots \quad (1)$$

$$V_2=\frac{f^2}{4\pi}\,.\,S_{12}.\,\frac{e^{-\lambda r}}{\lambda^2 r^3} \qquad \dots \quad (2)$$

where f^2 is the coupling constant, S_{12}, the tensor operator $=\frac{3(\bar{\sigma}_1.\bar{r})(\bar{\sigma}_2.\bar{r})}{r^2}-(\bar{\sigma}_1.\bar{\sigma}_2)$; σ_1 and $\bar{\sigma}_2$ denote the spin vectors of the two protons taking part in the collision; $\bar{r}$ denotes the vector joining the two protons; and $\lambda=1.3\times 10^{-13}\ \text{cm}^{-1}$.

METHOD OF CALCULATION

Born approximation has been used for the present calculations, the method being given by Ashkin and Wu (1948). As the above paper gives a fairly exhaustive treatment of the subject, some of the details of the method have been left out here.

According to Born approximation the scattering matrix of proton-proton scattering (without exchange), for a potential of the from $V=S_{12}J(r)$ is given by

$$S^{\prime}{}_{m_s{}'m_s{}''}(\theta,\phi)=C(\theta)\zeta_{m_s{}'m_s{}''}(\theta,\phi), \qquad \dots \quad (3)$$

where $$C(\theta)=(M/\hbar^2)\int r^2 J(r)\left[\frac{\sin Kr}{Kr}-\frac{3Kr\cos Kr-\sin Kr}{(Kr)^3}\right]dr, \quad \ldots (4)$$

where $$K=2k\sin\theta/2 \text{ and } k^2=\frac{4\pi^2 ME}{h^2} \quad \ldots (5)$$

M is the mass of the proton ; E is the energy of the incident proton in the centre of mass system and $\zeta_{m_s' m_s''}$ is defined by the equation

$$\zeta(\theta,\phi)\chi_{m_s''}=\sum_{m_s'=-1}^{+1}\zeta_{m_s' m_s''}(\theta,\phi)\chi_{m_s'}, \quad \ldots (5)$$

where $\chi_{m_s'}$ and $\chi_{m_s''}$ are the spin eigenfunctions

and $$\zeta(\theta,\phi)=\sigma_1.\sigma_2-3\sigma_1.\overline{n_0-n}\,.\,\sigma_2.\overline{n_0-n}$$

$\sigma_1, \overline{n_0-n}$ and $\sigma_2, \overline{n_0-n}$ being the components of σ_1 and σ_2 in the direction of momentum transfer. The values of the matrix elements $\zeta_{m_s' m_s''}(\theta,\phi)$ have been tabulated by Ashkin and Wu as also by Burhop and Yadav (1949). θ and ϕ are the polar and azimuthal angles of the scattered direction with respect to the direction of incidence. For proton-proton scattering, due to exchange, the scattering matrix will be modified to

$$S_{m_s' m_s''}(\theta,\Phi)=S'_{m_s' m_s''}(\theta,\phi)-S'_{m_s' m_s''}(\pi-\theta,\pi+\Phi) \quad \ldots (6)$$

the negative sign appearing because here scattering takes place in the triplet state only.

The triplet scattering cross section is then given by

$$\sigma_t=\frac{1}{3}\sum_{m_s'}\sum_{m_s''}|S_{m_s' m_s''}|^2 \quad \ldots (7)$$

The total cross section is therefore

$$\sigma=\frac{3}{4}\cdot\sigma_t=\frac{1}{4}\sum\sum|S_{m_s' m_s''}|^2 \quad \ldots (8)$$

This reduces to

$$\sigma=6\left(\frac{M}{\hbar^2}\right)\left\{C(\theta)^2+C(\pi-\theta)^2+C(\theta)C(\pi-\theta)\right\} \quad \ldots (9)$$

For the first potential (eq. 1), we have

$$C_1(\theta)=\frac{f^2}{4\pi}\cdot\left(\frac{M}{\hbar^2}\right)^2\cdot\frac{1}{\lambda}\left[\frac{3\lambda}{2K^2}-\left(\frac{3\lambda^2}{2K^3}+\frac{1}{2K}\right)\tan^{-1}\frac{K}{\lambda}\right] \quad \ldots (10)$$

Similarly for the second potential

$$C_2(\theta)=\frac{f^2}{4\pi}\left(\frac{M}{\hbar^2}\right)^2\cdot\frac{1}{\lambda^2}\left[\frac{\lambda^2}{2K^2}-\left(\frac{\lambda}{2K}+\frac{\lambda^3}{2K^3}\right)\tan^{-1}\frac{K}{\lambda}\right] \quad (11)$$

Accordingly, the differential cross sections are

$$\sigma_1=6.\frac{1}{\lambda^2}\left(\frac{M}{\hbar^2}\cdot\frac{f^2}{4\pi}\right)^2\left\{\left[\frac{3\lambda}{2K^2}-\left(\frac{3\lambda^2}{2K^3}+\frac{1}{2K}\right)\tan^{-1}\frac{K}{\lambda}\right]^2\right.$$

$$+\left[\frac{3\lambda}{2K'^2}-\left(\frac{3\lambda^2}{2K'^3}+\frac{1}{2K'}\right)\tan^{-1}\frac{K'}{\lambda}\right]^2+\left[\frac{3\lambda}{2K^2}-\left(\frac{3\lambda^2}{2K^3}+\frac{1}{2K}\right)\tan^{-1}\frac{K}{\lambda}\right]$$

$$\times\left[\frac{3\lambda}{2K'^2}-\left(\frac{3\lambda^2}{2K'^3}+\frac{1}{2K'}\right)\tan^{-1}\frac{K'}{\lambda}\right]\Bigg\} \quad \ldots \quad (12)$$

$$\sigma_2=6\cdot\frac{1}{\lambda^4}\left(\frac{M}{\hbar^2}\cdot\frac{f^2}{4\pi}\right)^2\Bigg\{\left[-\frac{\lambda^2}{2K^2}+\left(\frac{\lambda}{2K}+\frac{\lambda^3}{2K^3}\right)\tan^{-1}\frac{K}{\lambda}\right]^2$$

$$+\left[-\frac{\lambda^2}{2K'^2}+\left(\frac{\lambda}{2K'}+\frac{\lambda^3}{2K'^3}\right)\tan^{-1}\frac{K'}{\lambda}\right]^2+\left[-\frac{\lambda^2}{2K^2}+\left(\frac{\lambda}{2K}+\frac{\lambda^3}{2K^3}\right)\tan^{-1}\frac{K}{\lambda}\right]$$

$$\times\left[-\frac{\lambda^2}{2K'^2}+\left(\frac{\lambda}{2K'}+\frac{\lambda^3}{2K'^3}\right)\tan^{-1}\frac{K'}{\lambda}\right]\Bigg\} \quad \ldots \quad (13)$$

where $K'=2k\cos\theta/2$.

RESULTS

A preliminary analysis of the calculations reveal that σ_2 shows much less variation with angle than σ_1. The magnitude of f^2 is adjusted so as to give the best agreement between the theoretical and experimental results and its value is taken to be 2.642×10^{-17} ergs. cm. For proton energies of 345 Mev, the numerical values of the differential cross sections σ_1 and σ_2 are given below.

Angle in degrees	σ_1 in 10^{-27} cm^2/steradian	σ_2 in 10^{-27} cm^2/steradian
20	0365	4 061
25	01351	3.936
30	.04601	3.774
35	.05402	3.688
40	.05810	3.524
45	06241	3 399
50	06691	3.298
55	.07117	3.208
60	.07154	3.096
65	.07564	3.053
70	.0777	3 002
80	.08091	2.949
90	.08307	2.934

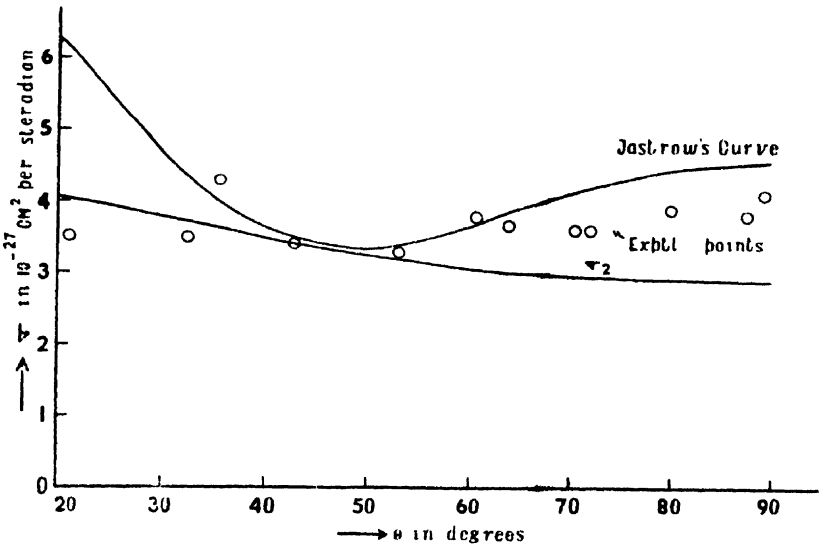

FIG. 1

In the accompanying graph σ_2 as well as Jastrow's curve for 345 MeV proton-proton scattering have been drawn. The circles given there denote the experimental values of Chamberlain *et al.* It may be seen that σ_2 fits more closely with the experimental points than Jastrow's curve. At present however, it does not appear appropiate to say that the potential V_2 chosen in this paper does give the exact law of interaction between protons at high energies of collision : the reason being that the experimental data are not sufficiently reliable and the calculations of the author have been performed in the non-relativistic approximation. All that can be said is that of the chosen potentials, V_2 can account for the angular isotropy of the differential cross sections to a great extent. Furthermore, it remains to be seen what justification the chosen potentials have in the light of a field theory.

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